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ON THE ANALYSIS OF RETAINED RESIDUAL VISCOPLASTIC PETROLEUM*

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A theory to determine the magnitude of the residual petroleum possessing a limit gradient in strata of inhomogeneous width is constructed in a hydrostatic approximation. In the general case the problem is reduced to a plane problem of nonlinear filtration and can be solved by known methods. In homogeneous and layered strata, the analysis of retained oil blocks results in boundary value problems of nonlinear filtration whose singularity is the existence of not only lines in the flow domain, but also of domains of a constant pressure gradient in absolute value, that equals the limit. Retained oil analysis schemes proposed earlier for homogeneous and layered-inhomogeneous strata /1—6/ need refinement on the basis of the general approach proposed.

1. Let us examine the concluding stage of water displacing viscoplastic petroleum from a stratum of width H whose properties, the permeability k and the limit gradient for the petroleum G, vary in width

$$k = k(z),\ G = G(z),\quad 0 \leqslant z \leqslant H \tag{1.1}$$

For simplicity, we assume that the permeability of the stratum decreases monotonically from the roof $z = 0$ to the floor $z = H$, and the limit gradient correspondingly increases monotonically so that $k'(z) \leqslant 0,\ G'(z) \geqslant 0$.

In the stage being considered, only water which has first displaced the petroleum from everywhere where the pressure gradient is above the limit, moves in the stratum. Neglecting the difference in the densities of the water and the petroleum, we assume the stratum sufficiently thin, the pressure distributed hydrostatically over to the width and able to be characterized by a two-dimensional distribution $p(x, y) = p(x, y, 0)$.

Depending on the magnitude of the pressure gradient, we take the following scheme for the formation of the residual petroleum blocks. The stratum is completely flushed by water in that domain of the stratum where the pressure gradient is larger, in absolute value, than the limit gradient corresponding to the least permeability, i.e., $|\nabla p(x, y)| > G(H)$. Let this domain be D_1. Where the pressure gradient is less than the limit corresponding to the greatest permeability, i.e., $|\nabla p(x, y)| < G(0)$, the immobile petroleum block occupies the whole width of the stratum (the domain D_3). In the domain D_2 where the pressure gradient is subject to the inequalities $G(0) < |\nabla p| < G(H)$, the whole width of the stratum is divided into two parts by the point $z = h(x, y)$ determined from the equation

$$G(h(x, y)) = |\nabla p(x, y)| \tag{1.2}$$

One part $0 \leqslant z \leqslant h(x, y)$, at each of whose points the condition $G(z) < |\nabla p(x, y)|$ is satisfied, is flushed by water. The other part of the stratum $h(x, y) \leqslant z \leqslant H$, for which $G(z) > |\nabla p(x, y)|$, is occupied by the immobile petroleum block. Here $h(x, y) = H$ in the domain D_1 and $h(x, y) = 0$ in the domain D_3.

For the scheme used, the water motion averaged relative to the width is described by the following system of equations:

$$\operatorname{div} \mathbf{w} = 0,\quad \mathbf{w} = -\frac{K(|\nabla p|)}{\mu}\nabla p \quad \mathbf{w} = \frac{1}{H}\int_0^{h(|\nabla p|)} \mathbf{v}(x, y, z)\,dz,\quad K(|\nabla p|) = \frac{1}{H}\int_0^{h(|\nabla p|)} k(z)\,dz \tag{1.3}$$

Here $\mathbf{w}$ and K are the effective velocity and permeability, and the width of the flushed part of the stratum $h(|\nabla p|)$ is determined from (1.2).

It follows from the system (1.3) that the equations of average water motion are equivalent to the equations of nonlinear filtration of an incompressible fluid

$$\operatorname{div} \mathbf{w} = 0,\quad \nabla p = -\frac{\Phi(w)}{w}\mathbf{w} \tag{1.4}$$

By the hodograph transformation

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$$dx + idy = e^{i\theta}\left(-\frac{dp}{\Phi(w)} + i\,\frac{d\psi}{w}\right) \tag{1.5}$$

the system (1.4) is transferred into a known linear system /7/

$$\frac{\partial p}{\partial\theta} = -\frac{\Phi^2(w)}{w\Phi'(w)}\frac{\partial\psi}{\partial w}, \quad \frac{\partial p}{\partial w} = \frac{\Phi(w)}{w^2}\frac{\partial\psi}{\partial\theta} \tag{1.6}$$

The specific form of the effective filtration law $\Phi(w)$ is determined by the form of the distributions $k(z)$ and $G(z)$ from (1.2) and (1.3).

2. Let us consider the examples. We hence assume that $G \sim k^{-1/2}$ in conformity with the known correlation for viscoplastic fluids.

Let the dependences $k(z)$ and $G(z)$ be

$$k(z) = k_0(1 + z / z_0)^{-2}, \; G(z) = G_0(1 + z / z_0) \tag{2.1}$$

Here z_0 is a certain parameter and $G_0 = G(0)$. We obtain the following effective filtration law from (1.2) and (1.3):

$$\Phi(w) = \frac{\mu(w+\lambda_0)}{K_0}, \quad \lambda_0 = \frac{k_0 z_0}{\mu H} G_0, \quad K_0 = \frac{\mu\lambda_0}{G_0} \tag{2.2}$$

Therefore, for the permeability and limit gradient distributions in the form (2.1) the problem of seeking the petroleum block in an average formulation reduces to the known problem with a limit gradient for a homogeneous fluid.

The validity of the following correspondences

$$k(z) = k_0\,\mathrm{ch}^{-2}(z / z_0), \quad G(z) = G_0\,\mathrm{ch}(z / z_0), \quad \Phi(w) = \mu(w^2 + \lambda_0^2)^{1/2} / K_0 \tag{2.3}$$

can be seen analogously.

If the dependence $k(z)$ allows of the parametric representation

$$\frac{dz}{dh} = \frac{k_0\alpha}{\mu}\left(\mathrm{sh}\frac{h}{z_0}\,\mathrm{ch}\frac{h}{z_0}\right)^{\alpha-1}, \quad k(h) = \frac{k_0}{\mathrm{ch}^{2\alpha}(h/z_0)}, \quad G(h) = \frac{G_0}{k_0^{1/2}}\mathrm{ch}^{\alpha}\frac{h}{z_0}$$

then the corresponding effective filtration law has the form

$$\Phi(w) = \mu(w^{2/\alpha} + \lambda_0^{2/\alpha})^{\alpha/2} / K_0 \tag{2.4}$$

Letting the parameter α tend to zero, we obtain

$$\Phi(w) = G_0, \; w < \lambda_0, \; \Phi(w) = \mu w / K_0, \; w > \lambda_0 \tag{2.5}$$

which corresponds to a homogeneous stratum of permeability k_0.

In all the examples, the relationships for the effective filtration law are valid for velocities less than $\lambda_H = \mu^{-1} K_H G(H)$, hence $\Phi(w) < G(H)$. For high velocities, the stratum is completely flushed by water, and the effective permeability ceases to vary as the intensity of the motion changes, and the corresponding filtration law for the average motion turns out to be linear in the high velocity domain

$$\Phi(w) = \frac{\mu w}{K_H}, \quad K_H = \frac{1}{H}\int_0^H k(z)\,dz, \quad |\nabla p| \geqslant G(H)$$

In cases when the total motion intensity is not large, the flushed zones are localized completely near the boreholes. If their influence on the process of petroleum block formation can be neglected, then the average motion in the whole stratum is described by a nonlinear filtration law of the form (2.2)—(2.5). Formally, this corresponds to the asymptotic $H \to \infty$. Numerous solutions constructed earlier for nonlinear filtration problems can here be used to estimate the petroleum block sizes.

We obtain an estimate for the volume of the flushed part of the stratum. The latter evidently equals

$$V = \iint_D h(x, y)\,dx\,dy = \iint_\Delta h(w)\,J(w, \theta)\,dw\,d\theta \tag{2.6}$$

where D is the flow domain in the physical plane, Δ is its corresponding domain in the hodograph plane, and $J(w, \theta)$ is the Jacobian of the transition. Relying on (1.5) and (1.6), we convert (2.6) to the form

$$V = \iint_\Delta \frac{h(w)}{w\Phi(w)}\left[\left(\frac{\partial\psi}{\partial\theta}\right)^2\frac{\Phi(w)}{w^2} + \left(\frac{\partial\psi}{\partial w}\right)^2\frac{\Phi^2(w)}{w\Phi'(w)}\right]dw\,d\theta \tag{2.7}$$

Since $h(w) / w\,\Phi(w) \leqslant MH$, where $M = \mu / (k_0 G_0^2)$, the upper bound for the volume of the flushed part of the stratum hence follows from (2.6):

$$V \leqslant MH \int_l \psi \, dp$$

where l is the boundary of the flow domain in the hodograph plane. In particular, we have for an equal-intensity source-sink system in an unbounded stratum

$$V \leqslant \mu Q \mid p_2 - p_1 \mid H \,/\, (k_0 G_0^2)$$

where p_2 and p_1 are the pressure on the borehole and on the supply contour, respectively.

3. Let us consider the case of a homogeneous stratum $k = \text{const}$. For such a stratum $G(0) = G(H) = G$, and the width of the part of the stratum flushed by the water is $h(|\nabla p|)$, and it and the effective permeability $K(|\nabla p|)$ become piecewise-constant functions of the pressure gradient

$$h(|\nabla p|) = 0, \quad K(|\nabla p|) = 0, \quad |\nabla p| < G$$
$$h(|\nabla p|) = H, \quad K(|\nabla p|) = k, \quad |\nabla p| > G$$

The deduction was hence made earlier /1–6/ that when the pressure gradient reached the value G, equal to the limit, the width of the flushed layer changed from zero to the full width of the stratum by a jump on a certain line of the physical plane. This corresponds to an effective filtration law of the form:

$$\Phi(w) = \mu w / k, \quad w > \lambda; \quad 0 \leqslant \Phi(w) \leqslant G, \quad w = 0, \quad \lambda = kG / \mu$$

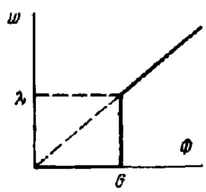

Fig.1

(the discontinuous law of M. G. Alishaev et al. /1/). However, the passage to the limit from the flow scheme described above in strata with continuously varying permeability to flows in homogeneous strata results in the deduction that the condition that the abosolute value of the pressure gradient equals the limit is not satisfied on a line (the boundary of the petroleum block), but in a domain in which the width of the flushed layer $h(x, y)$ is a continuous function of the water flow. As the effective filtration velocity changes from zero to λ the width of the flushed layer runs through all values between 0 and H. The corresponding effective filtration law has the form (Fig.1)

$$\Phi(w) = \mu w / k, \quad w > \lambda; \quad \Phi(w) = G, \quad 0 < w \leqslant \lambda, \quad 0 \leqslant \Phi(w) < G, \quad w = 0 \tag{3.1}$$

In contrast to the discontinuous law, the filtration law (3.1) permits consideration of the flow even in a domain of velocities w less than λ.

Therefore, as blocks of residual petroleum form in homogeneous strata, the whole flow domain in the physical plane decomposed into three subdomains in the general case: the domain D_1 of a completely flushed stratum; the domain D_2 of a partially flushed stratum in which the absolute value of the pressure gradient is constant and equal to the limit; the domain D_3 in which the petroleum block occupies the whole width of the stratum and there is no water motion.

We have for each domain

$$\Delta p(x, y) = 0, \quad h(x, y) = H, \quad (x, y) \in D_1, \quad |\nabla p(x, y)| = G, \quad \operatorname{div}(h(x, y) \nabla p / G) = 0, \quad (x, y) \in D_2 \tag{3.2}$$
$$w(x, y) = 0, \quad h(x, y) = 0, \quad (x, y) \in D_3$$

The solutions merge on the domain boundaries by means of conditions on the continuity of the pressure, the flow, and the width $h(x, y)$.

On going over to the hodograph plane (w, θ), the domain D_1 is mapped into the domain Δ_1 in the half-plane $w > \lambda$, the domain D_2 into the domain Δ_2 in the strip $0 < w < \lambda$, and the domain D_3 into the line segment $w = 0$. In the corresponding domains of the hodograph plane, (3.2) take the form

$$\frac{\partial \psi}{\partial w} = -\frac{k}{\mu w} \frac{\partial p}{\partial \theta}, \quad \frac{\partial p}{\partial w} = \frac{\mu}{k w} \frac{\partial \psi}{\partial \theta}, \quad (w, \theta) \in \Delta_1, \quad \frac{\partial \psi}{\partial w} = 0, \quad \frac{1}{G} \frac{\partial p}{\partial w} = \frac{1}{w^2} \frac{\partial \psi}{\partial \theta}, \quad (w, \theta) \in \Delta_2 \tag{3.3}$$

from which we have the solution

$$\psi = f(\theta), \quad p(w, \theta) = -\frac{G f'(\theta)}{w} + \varphi(0) \tag{3.4}$$

for the constant pressure gradient domain Δ_2, where $f(\theta)$ and $\varphi(0)$ are unknown functions. The variables (w, θ) and the physical coordinates (x, y) are interconnected for this domain by the relationships (here and henceforth $z = x + iy$)

$$z(w, \theta) = z(\lambda, \theta) + e^{i\theta} f'(\theta) \, (1 / w - 1 / \lambda) \tag{3.5}$$

It follows from (3.5) that for $f'(\theta) \neq 0$ the domain Δ_2 in the physical plane corresponds to the domain in which the streamlines are straight lines, the pressure varies linearly along them, and the effecitive velocity w and the flushed part of the stratum h are determined by the expressions

$$w = (1/\lambda + |z(w, \theta) - z(\lambda, \theta)| / f'(\theta))^{-1}, \quad h = Hw / \lambda \tag{3.6}$$

If $f'(\theta) = 0$, then the corresponding part of the domain Δ_2 is mapped in the physical plane into a line that is a segment of a streamline. The fluid flow at these points is directed along the tangent to the line $|\nabla p| = G$, for which the width of the flushed part of the stratum $h(x, y)$ changes by a jump from 0 to H when it is crossed.

In other words, the problem formulation used earlier with the jumplike change in the flushed width, turns out to be a corollary of the formulation elucidated here if and only if the unknown boundary is a streamline. A retrospective analysis shows that the situation is precisely this in almost all problems solved earlier, and in particular, within the framework of the refined formulation, all solutions presented in /1— 3/, are meaningful with the exception of the solutions of Figs.7.19 and 7.20 in /3/. Meanwhile, the refined formulation even permits investigation of such flow configurations as did not previously allow of examination.

Let us note that the assumption of constancy of the absolute value of the pressure gradient, equal to the limit in a domain rather than on a line, was used in /8/, however, it was simultaneously assumed that there is no flow in this domain.

4. Let us present an example of analyzing in immobile petroleum block for the flow to a single borehole located within the supply contour, along which constant pressure is given. Let the supply contour be a circle of radius R, the borehole be located eccentrically at a distance ρ from the center of the supply contour, and its debit equal to Q per unit width of the stratum.

Leaving a detailed parametric analysis of the problem aside, we note that a whole series of possible pictures of the pertroleum block locations occurs during solution which result in different boundary value problems. For all the cases, the flow in the constant pressure gradient domain possesses still another property in addition to those noted earlier: the isobars are concentric circles, and the streamlines are segments of rays issuing from the origin.

Let us examine the range of parameters in greater detail when the condition $0 < w < \lambda$ is satisfied everywhere on the supply contour. In this case the domain D_1 of the completely flushed stratum is separated from the supply contour by the constant pressure gradient domain D_2 in which part of the stratum width is occupied by the immobile petroleum block.

For the case under consideration, the solution of the problem can be obtained as follows. We select the coordinate system (x, y) so that the origin coincides with the center of the supply contour, and the borehole has the coordinates $x = -\rho,\ y = 0$. Using the solution (3.4) in the constant gradient domain, we obtain that the problem in the hodograph plane reduces to the following boundary value problem for a stream function in a half-strip:

$$\frac{1}{w}\frac{\partial}{\partial w}\left(w\frac{\partial\psi}{\partial w}\right) + \frac{1}{w^2}\frac{\partial^2\psi}{\partial\theta^2} = 0, \quad 0 \leqslant \theta \leqslant \pi, \quad \lambda \leqslant w < \infty, \quad \psi = \frac{\theta}{2\pi}Q, \quad w \to \infty, \quad 0 \leqslant \theta \leqslant \pi \tag{4.1}$$

$$\lambda\frac{\partial\psi}{\partial w} = \frac{\partial^2\psi}{\partial\theta^2}, \quad w = \lambda, \quad 0 \leqslant \theta \leqslant \pi, \quad \psi = 0, \quad \lambda \leqslant w < \infty, \quad \theta = 0, \quad \psi = Q/2, \quad \lambda \leqslant w < \infty, \quad \theta = \pi$$

The solution of (4.1) is found in elementary functions

$$\psi(w, \theta) = \frac{Q}{2\pi}\theta + A\frac{\lambda}{w}\sin\theta \tag{4.2}$$

where the unknown constant $A = -2\lambda\rho$ is determined from the continuity condition for the physical coordinates upon crossing the line $w = \lambda$. The equation for this latter in the physical plane has the form

$$z(\lambda, \theta) = \frac{e^{i\theta}}{\lambda}\left(\frac{Q}{2\pi} - 2\lambda\rho\cos\theta\right) \tag{4.3}$$

From the relations (4.2) and (4.3) we obtain an expression for the width of the flushed part of the stratum in the constant pressure gradient domain, as well as the ratio between the volume of the immobile petroleum block and the volume of the stratum

$$h = \frac{H}{\lambda r}\left(\frac{Q}{2\pi} - 2\lambda\rho\cos\theta\right), \quad r = (x^2 + y^2)^{1/2}, \quad \eta = \left(1 - \frac{Q}{2\pi R\lambda}\right)^2 + 2\left(\frac{\rho}{R}\right)^2$$

It is seen from (4.2) and (4.3) that the solution obtained is valid under the following constraints on the problem parameters:

$$\frac{\rho}{R} \leqslant \frac{Q}{4\pi R\lambda} \leqslant \frac{1}{2} - \frac{\rho}{R}$$

The results of computing one of the variants of this case for $Q / 2\pi\lambda R = 0.5$ and $\rho / R = 0.2$ are presented in Fig.2, where the boundaries of the constant pressure gradient domain and the stream lines within this domain are shown above, and the section of the stratum along the x axis is below. The domain occupied by the immobile petroleum block is shaded.

5. Let us now consider a layered stratum with piecewise-constant distribution of the permeability (and, therefore, a limit gradient for petroleum) over the width. Let the number of interstratifications of constant permeability k_i be n, let each possess the width H_i and the limit gradient G_i, where $k_1 > k_2 > \ldots > k_n$ and $G_1 < G_2 < \ldots < G_n$.

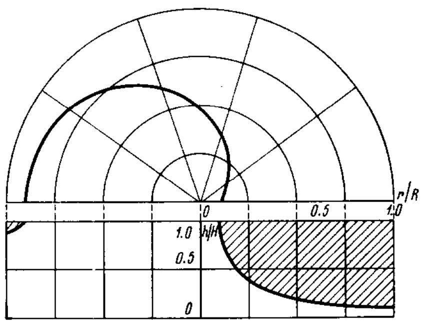

Fig.2

Again considering the layered stratum as the limit case of strata with continuous permeability distribution along the width, we obtain that the formulation of the problem of determining the residual petroleum blocks for layered strata is analogous to that for homogeneous strata, and has the form

$$\Delta p(x, y) = 0, \quad G_j < |\nabla p(x, y)| < G_{j+1}$$

$$K(x, y) = K_j = H^{-1} \sum_{i=1}^{j} k_i H_i, \quad h(x, y) = h_j = \sum_{i=1}^{j} H_i$$

$$j = 1, 2, \ldots, n, \quad (x, y) \in D_1^{(j)}$$

$$|\operatorname{grad} p(x, y)| = G_j, \quad \operatorname{div}(K(x, y) \operatorname{grad} p(x, y)) = 0$$

$$K(x, y) = K_{j-1} + (k_j / H)\, h(x, y), \quad K_{j-1} \leqslant K(x, y) \leqslant K_j$$

$$h_{j-1} \leqslant h(x, y) \leqslant h_j, \quad j = 1, 2, \ldots, n, \quad (x, y) \in D_2^{(j)}$$

$$w(x, y) = 0, \quad h(x, y) = 0, \quad (x, y) \in D_3$$

On the domain boundaries, the solutions are again merged by the continuity conditions on the pressure, the flow, and the width of the flushed part of the stratum. Therefore, a domain constant pressure gradient $D_2^{(i)}$, equals to the limit, and the domain $D_1^{(i)}$ of constant width of the flushed part $h(x, y) = h_i$ correspond to each substratum in the flow domain in the formation of residual petroleum blocks. The whole flow domain is now divided into $2n + 1$ subdomains.

The effective filtration law for layered strata is a piecewise-linear function

$$\Phi(w) = \mu w / K_i, \quad \lambda_i < w < \Lambda_i, \quad \Phi(w) = G_i, \quad \Lambda_{i-1} < w < \lambda_i, \quad 0 \leqslant \Phi(w) \leqslant G_1, \quad w = 0$$

$$\lambda_i = \frac{K_i}{\mu} G_i, \quad \Lambda_i = \frac{K_i}{\mu} G_{i+1}, \quad \Lambda_0 = 0, \quad \Lambda_n = \infty, \quad i = 1, 2, \ldots, n$$

The characteristic feature of this law is the presence of n constancy sections to each of which a domain $D_2^{(j)}$, where the streamlines are straight lines and the equations of motion are integrated, corresponds in the physical plane. The solution has the form (3.4), (3.5), where $w = \lambda_i$ must be substituted instead of $w = \lambda$.

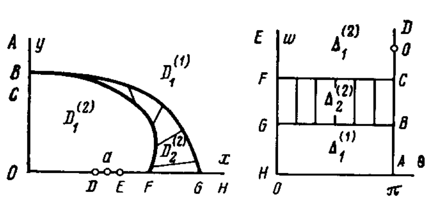

Fig.3

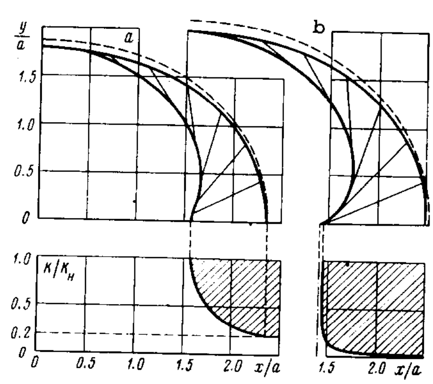

Fig.4

6. Let us determine the shape of the residual petroleum block for a flow produced by a source and sink of equal intensity Q in a boundless stratum. Let the stratum consist of two substrata, and let the limit gradient for the petroleum in the more permeable substratum equal zero $G_1 = 0$, i.e., the effective filtration law has the form

$$\Phi(w) = \mu w / K_1,\ 0 < w < \Lambda_1, \quad \Phi(w) = G_2,\ \Lambda_1 < w < \lambda_2, \quad \Phi(w) = \mu w / K_2,\ \lambda_2 < w < \infty$$

The first quadrant is the symmetry element of the problem in the physical plane (x, y) Using the fact that the solution is known in the constant pressure gradient domain, and going over to the hodograph plane, we arrive at the following problem for the stream function (Fig. 3):

$$w \frac{\partial}{\partial w}\left(w \frac{\partial \psi}{\partial w}\right) + \frac{\partial^2 \psi}{\partial \theta^2} = 0, \quad 0 \leqslant \theta \leqslant \pi;\ 0 \leqslant w \leqslant \Lambda_1,\ \lambda_2 \leqslant w < \infty, \quad \psi(w, \theta) = \frac{Q}{2\pi}\theta, \quad w \to \infty, \quad 0 \leqslant \theta \leqslant \pi \tag{6.1}$$

$$\psi(w, \theta) = 0,\ w \to 0,\ 0 \leqslant \theta \leqslant \pi, \quad \psi(w, \theta) = Q/2,\ w_3 < w < \infty,\ \theta = \pi, \quad \partial\psi / \partial w = 0,\ 0 < w < w_3,\ \theta = \pi$$

$$\frac{\lambda_2}{K_2}\left(\frac{\partial \psi}{\partial w}\right)_{w=\lambda_2} - \frac{\Lambda_1}{K_1}\left(\frac{\partial \psi}{\partial w}\right)_{w=\Lambda_1} = G_2\left(\frac{\partial^2 \psi}{\partial \theta^2}\right)_{w=\Lambda_1}\left(\frac{1}{\lambda_2} - \frac{1}{\Lambda_1}\right), \quad \psi(\Lambda_1, \theta) = \psi(\lambda_2, \theta),\ 0 \leqslant \theta \leqslant \pi$$

The parameter w_3 in (6.1) is determined from the additional condition that the distance between the source and sink in the physical plane was equal to the given value. After having solved problem (6.1), by integrating the formulas for the transition to the physical plane (x, y), we find the boundary of the constant pressure gradient domain, then the width of the flushed part of the stratum, and we thereby determine the shape of the immobile petroleum block.

The solution is constructed numerically in dimensionless variables. The quantities a and Q/a are selected as length and velocity scales, and the solution hence depends on two dimensionless parameters

$$\varepsilon = \frac{\pi a K_2 G_2}{\mu Q}, \quad \delta = \frac{k_1 H_1 + k_2 H_2}{k_1 H_1}$$

Here ε is a dynamic parameter and δ characterizes the degree of inhomogeneity of the stratum.

The results of computing two versions corresponding to the parameters $\varepsilon = 0.4$ and $\delta = 5(a)$ and $\delta = 100$ (b) are presented in Fig.4 as solid lines. Shown for comparison by dashed lines are the residual petroleum blocks obtained within the framework of an approximate approach when the constant pressure gradient domain is replaced by lines. The problem in such a formulation is solved in /4/ by potential theory methods. It follows from the comparison that the petroleum block boundaries obtained in /5/ can be the lower estimate for the determination of the magnitude of the immobile petroleum.

Analysis of the example for $\delta = 100$, when the effective width of the more permeable substratum is small compared to the effective width of the second substratum, shows that (Fig.4b) when the pressure gradient G_2 is reached, the stratum at once is blocked almost completely by the immobile petroleum. It is interesting to make a comparison with the limit case $\delta = \infty$ corresponding to a homogeneous stratum. The constant pressure gradient domain in this case degenerates into a line separating the domain of the completely flushed stratum from the domain in which the strata are occupied by immobile petroleum over the whole width. Such a problem is solved in /1/, where an expression is obtained for the boundary of the dead zone (or the petroleum blocks in the treatment used here). This solution is shown by dash-dot lines in Fig.4a. As should have been expected, the results for $\delta = 100$ and $\delta = \infty$ practically agree. This latter fact permits using the solution obtained in /1/ to estimate the immobile petroleum blocks in two-layered strata with small effective width of the more permeable substratum.

REFERENCES

1. ALISHAEV M.G., VAKHITOV G.G., GEKHTMAN M.M., and GLUMOV I.F., On some features of the filtration of Devonian stratum petroleum at lowered temperatures. Izv. Akad. Nauk SSSR, Mekhan. Zhidk. Gaz, No.3, 1966.

2. ALISHAEV M.G., BERNADINER M.G., and ENTOV V.M., Influence of the limit gradient on the loss of petroleum during displacement by water. In: Questions of Nonlinear Filtrations and Petroleum-Gas Elimination during the Development of Petroleum and Gas Deposits. Moscow, Izd. Inst. Geologii i Razrabotki Goriuchikh Iskopaemykh, 1972.

3. BERNADINER M.G., and ENTOV V.M , Hydrodynamic Theory of Filtration of Anomalous Fluids, Moscow, "Nauka", 1975.

4. KOTLIAR L.M. and SKVORTSOV E.V., On the filtration of a viscoplastic fluid to a sink in a curvilinear stratum. Dokl. Akad. Nauk SSSR, Vol.209, No.5, 1973.

5. ENTOV V.M., MALAKHOVA T.A., PANKOV V.N., and PAN'KO S.V., Calculation of the limit-equilibrium of retained viscoplastic oil extracted from a nonuniform stratified layer by water, PMM, Vol.44, No.1, 1980.

6. KOTLIAR L.M. and SKVORTSOV E.V., Plane Stationary Problems of Filtration of a Fluid with Initial Gradient. Izd. Kazansk. Univ, 1978.

7. KHRISTIANOVICH S.A., Motion of ground water motion not obeying Darcy's law, PMM, Vol.4, No.1, 1940.

8. FEDOROV A.V. and FOMIN V.M., Mathematical model of gas motion in coal strata for a nonlinear filtration law. In: Numerical Methods of the Mechanics of a Continuous Medium, Vol.4, No.5, Novosibirsk, Izd. Vychisl. Tsentr Novosibirsk. Itdel. Akad. Nauk SSSR, 1973.

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